Vladimir A. Bragin, analyst
Vladimir V. Osakovsky, PhD
Senior expert

Institute for Open Economy
Moscow, 2003.
Department of labor economics

Estimation of natural unemployment rate in the Russian Federation: empirical analysis of Russian labor market efficiency

JEL codes: J11, J21, J61, J64, O15, O18, R23

Summary

To specify the problem of unemployment in Russia, we estimate the natural rate of unemployment by consecutively estimating the optimal size of the labor force and the optimal employment. For estimation of the optimal values we used modified Hodrick-Prescott filter technique. The results show that the natural rate of unemployment in Russia during 1994-1997 was stable around 12-12,5%, and decreased to 8,1% by 2003. Moreover, before 1998 the actual unemployment was significantly lower than its natural rate and today practically equals it.

1. Introduction

Unemployment is one of several new economic phenomena, brought into the lives of ordinary Russians by the liberalization of the economy at the end of the 20-th century. However, despite numerous catastrophic forecasts, very popular at the beginning of the reforms, throughout the whole period of reformations and until today unemployment has not become the dominant factor on the Russian labor market. Although within several years Russian GDP fell by almost 50%, total employment fell by less than 18 %, with the unemployment rate rising to at most 13-14 % of the labor force. By 2003 the unemployment rate declined to just 8 % of the labor force.

Major explanation to this phenomenon is the fact, that the initial job market response to recession was a reduction of real wages, instead of the reduction of employment. This was largely induced by soft monetary policy of the Russian Central Bank which led to hyperinflation in the beginning of 1990-s'.

Another major trend on the job market over the last decade was a reduction of the size of the labor force. According to statistics, from 1992 to 2001, a working-age population out of the labor force has increased by more than 6,6 million, and labor force participation rate fell by over 6 %. This is equivalent to removal of more than 7,7 million from the labor market.

Another major reason for a relatively low unemployment rate in Russia is the beginning of labor supply and demand balancing. At the beginning of reforms, characteristic feature of the Russian labor market was a sharp labor deficit. According to a number of estimates [2, 3, 6] the problem of unequal growth of open vacancies and labor force was observed in RSFSR since 1970. The problem reached critical levels in second half of 80-s', when the number of open vacancies grew to over 50 % of population not engaged in the economy and to more than 10 % of the number of employed. In this situation, initial adaptation of the labor market to reforms was closure of vacancies. As a result, in a few years, number of vacancies fell to a 1-2 % level of the total number of employed.

On the other hand, some researchers [1, 4] note that labor deficit in the Soviet economy existed only because the employment rate in pre-reform Russia exceeded its optimal level by as much as 15%. In this respect, as one might have expected, with the beginning of reforms, companies could have started massive reduction of «unnecessary» workers, which would result in significant or even catastrophic growth of unemployment. Notwithstanding the fact that this scenario failed to realize, the reduction of excessive employment is yet another major factor, which rendered and continues to render influence on the Russian labor market.

Overall, throughout the whole period of reforms and up until today, the labor market in Russia has been influenced by a wide range of factors, which have an extremely versatile and complex influence (reduction of the labor force, recession of the economy, change of employer behavior towards «unnecessary» workers etc.).

In this respect, considering social and political importance of the labor market, the goal of this research is the estimation of the unemployment rate, which would exist in Russia under the conditions of a long-term market balance, i.e. the rate corrected on short-term effects of recession, and effects of overcoming «heritage of planning economy». Estimation of such level can give more objective view on the unemployment problem in the country, and can also help to determine most effective set of policy measures.

Using the standard terminology, the goal of this research is estimation of the natural rate of unemployment in Russia, or the rate of unemployment, which underlying reasons, according to Friedman [8] are «natural» (demographic, institutional, social), instead of monetary or cyclical factors. This is the rate of unemployment, which can be achieved in the absence of short-term effects due to unexpected changes in inflation and/or productivity.

Given the importance of the subject, there is surprisingly no research on natural unemployment in Russia in existing literature. A number of the researchers [1, 2, 3, 7] make comments on the general structure of existing unemployment, without undertaking any attempts to estimate quantitatively and qualitatively the level of conformity of the actual rate to the optimal or natural rate.

2. The model

There are several techniques to estimate the natural rate of unemployment (as also the «natural» levels of others variables). For example, the Agency of economic planning of Japan uses historical averages. Other researchers, under the natural level take a level with all other factors of production completely utilized. The third alternative of definition is the unemployment rate with stable inflation.

Given wide range of factors influencing labor market, we use an alternative approach, by estimating natural unemployment rate through consecutive estimation of major constituents of this level, i.e. optimal level of employment and optimal size of the labor force.

One of the positive moments of such approach is that in addition to the estimation of the target parameter, we also get other interesting variables, such as optimal level of employment and long-term size of the labor force.

Technically, our method represents a combination of the first and third estimation method. Under natural employment rate we take employment rate at the stable inflation taking into account economy wide recession. At the same time, under the optimal size of the labor force we take its level in the absence of demographic changes with stable monetary incomes.

2.1. Model of employment

Theoretical background for the optimal employment rate estimation is positive correlation between employment and output, or Okun's law [17], and negative dependence of unemployment on inflation first formulated by Phillips [18], and later modified by a number of researchers [8, 18, 19]. In addition to this, in the analysis we also include real wages.

The procedure includes estimation of the regression, with deviation of the actual labor force size from its optimal level as a dependent variable, and real wages, rate of inflation and cumulative demand as independent variables. Successful estimation of this regression will give an estimate of the optimal level of employment.

Phillips curve estimation on the Russian data provides significant difficulty given deep changes of monetary regimes over the past decade. Such changes can make the influence of inflation on unemployment unequivocal. Besides, the time period under consideration is very short for estimation of the long-term «normal» level. In this respect, the effect of changes in inflation can have quite ambiguous influence on the labor market depending on time and level of price dynamics.

As an attempt to deal with this problem we outline 3 major inflationary periods since the beginning of reforms in Russia: hyperinflation of 1992-1994, period of exchange rate corridor (1994-1998) and floating rate period which began after the

1998 financial crisis. To differentiate between these periods we include 2 dummy variables: one for the hyperinflation periods at the beginning of reforms (periods from the beginning of a time trend with quarterly inflation rates exceeding 10 %) and the second for all periods, after 3'd quarter of 1998.

The model can be outlined as follows:

$$\Delta E_t = \alpha\left(E^*_{t-1} - E_{t-1}\right) + \beta_1 \Delta Y_t + \beta_2 \Delta \pi_t + \beta_3 \Delta \pi_{t-1} + \beta_4 \Delta w_t + \beta_5 \Delta w_{t-1} + \gamma_1 D_1 + \gamma_2 D_2 + \varepsilon_t \quad (1)$$

Where:

π_t - inflation,

Y_t - cumulative demand (GDP),

w_t - real wages,

$D_{1,2}$ - inflationary period dummy variable,

E_t - employment,

E_t^* - optimal employment.

If E_t exceeds (lower than) E_t^* - the economy has an excess (insufficient) employment, which creates a tendency for its reduction (increase).

In the presence of the data for E_t^*, or given the assumption of its constant level over the whole period, one could estimate all unknown parameters (α, β and γ) by a simple OLS. However, in our case, the assumption of the constant optimal level does not apply. Therefore, it is necessary to use a different technique which allows simultaneous estimation of E_t^* and all other parameters.

In this research we use the technique applied by Hirose and Kamada [14], who simultaneously estimate Phillips curve and potential (natural) GDP level on the assumption that this parameter changes smoothly over the whole time interval. In their work, potential output level is a moving average of a GDP trend, corrected on inflation, which is estimated using a modified version of a Hodrick-Prescott (HP) filter [15].

Hodrick-Prescott filter is a time series technique, which estimates the series x_t^{HP}, which minimizes the following objective function:

$$\sum_{t=1}^{T}(x_t - x_t^{HP})^2 + \lambda \sum_{t=2}^{T-1}(\Delta x_{t+1}^{HP} - \Delta x_t^{HP})^2, \quad (2)$$

where λ is a parameter of smoothness of x_t^{HP} changes, which is a moving average for x_t series.

However, since x_t^{HP} is a simple moving average for x_t, its correlation with other independent variables is practically absent. Therefore, x_t^{HP} can not be considered as a good estimate for the natural unemployment level, given the assumption that it is influenced by a number of external factors (inflation, etc.). The applied modification of the HP filter deals exactly with this problem.

The main idea of the technique is that we filter not E_t series, but a new series g_t, which is derived from the model of employment (1), and equals to:

$$g_t = \{E_t - (1+\alpha)E_{t-1} - \beta_1 \Delta y_t - \beta_2 \Delta \pi_t - \beta_3 \Delta \pi_{t-1} - \beta_4 \Delta w_t - \beta_5 \Delta w_{t-1} - \gamma_1 D_1 - \gamma_2 D_2\}/\alpha \quad (3)$$

In this case, the objective function is defined as:

$$W(\alpha, \beta_1, .., \beta_5, \gamma_1, \gamma_2, E_1^N ... E_{T-1}^N) = \sum_{t=1}^{T} \{g_t - \alpha E_{t-1}^N)\}^2 + \lambda \sum_{t=2}^{T-1} (\alpha \Delta E_{t+1}^N - \alpha \Delta E_{t-1}^N)^2 \quad (4)$$

To obtain the values of all necessary parameters of the objective function we use the following approach. First of all, the parameters of the model (1) (α, $\beta_1, ..., \beta_5$, γ_1 and γ_2) are fixed at the arbitrary values, and, given them, we solve for T unknowns (E_1^N ,…, E_T^N) by HP filtering the g_t series. After that, we choose optimal values of other parameters, which minimize the objective function W. In this procedure, for a smoothness parameter λ we use the standard value for quarterly data (λ = 1600). The estimation is followed by standard tests for statistical significance of obtained parameters.

2.2. Labor force model

Since the beginning of reforms, the size of the Russian labor force had been under the influence of structural and institutional factors arising from transition to market economy (see Kapelyushnikov, 2001). In particular, Kapelyushnikov points at the reduction of employment opportunities for women with children and pensioners, which led to «more rational model of labor distribution among economic activities, similar to those of more mature economies». In other words, transition of the economy led to shifting from an abnormally high level of labor force participation rate of the Soviet period to a much lower level of a market economy.

In this respect, we suggest, that the real value of the optimum level of labor force participation during the transition period is close to the trend of this parameter changes.

Among other factors influencing the labor force participation, we can also point at the demographic factors (population changes as changes of number of the potential labor market participants), and also the real incomes of the population. The latter is included in the analysis given strong theoretical background from standard economic theory, according to which growth of real incomes increases opportunity costs of leisure, thus leading to an increased of supply of labor.

Taking into account all of these factors, the optimal level of labor force participation is defined as the trend of labor force participant quantity corrected on population dynamics and the changes in real income.

Thus, the model of economic activity can be defined as follows:

$$A_t = A_t^* + \eta \Delta POP_t + \mu \Delta w_t + \varepsilon_t, \quad (5)$$

Where:

ΔPOP_t –dynamics of the population in the age from 15 to 72 years, or potential labor supply,

Δw_t - real income dynamics,

A_t and A_t^* - actual and optimal size of the labor force, respectively.

Using technique applied for the estimation of the optimum level of employment, temporary series for the labor force model (5), is set as follows,

$$h_t = A_t - \eta \Delta POP_t - \mu \Delta w_t, \quad (6)$$

and objective function as,

$$H(\eta, \mu, A_1^* \ldots A_T^*) = \sum_{t=1}^{T} \{h_t - A_t^*\}^2 + \lambda \sum_{t=2}^{T-1} (\Delta A_{t+1}^* - \Delta A_t^*)^2. \quad (7)$$

Using the same technique, and same initial parameters, as for model of employment we estimate T unknowns A_1^* ,…, A_T^*.

3. Results

We used quarterly data from the beginning of 1994 till the first quarter of 2003. For the model of employment we used data on the number of people employed in all sectors of the economy, CPI and real GDP dynamics. For the labor force model we used data on the size of the Russian labor force, population in the age from 15 till 72 years, and real income dynamics. All data are provided by the Russian statistical agency – Goskomstat.

First, we estimate regressions for the models outlined above and undertake measures for its possible restriction. Second, we conduct comparative analysis of natural and actual levels, and explore their important characteristics. In the end, we use the obtained natural levels of employment and labor force to calculate the natural rate of unemployment.

3.1. Optimal employment

The results of the employment model estimation defined by equations 3 and 4 are given in table 1.1.

$$\Delta E_t = \alpha\left(E_{t-1}^* - E_{t-1}\right) + \beta_1 \Delta Y_t + \beta_2 \Delta \pi_t + \beta_3 \Delta \pi_{t-1} + \beta_4 \Delta w_t + \beta_5 \Delta w_{t-1} + \gamma_1 D_1 + \gamma_2 D_2 + \varepsilon_t$$

Table 1.1

	Coefficient	Standard error	t – stat
α	0,1894	0,0819	2,3135
β_1	0,0427	0,0096	4,432
β_2	0,0144	0,0091	1,5823
β_3	-0,014	0,0075	-1,875
β_4	0,0122	0,0111	1,1047
β_5	0,0134	0,0138	0,9722

γ_1	0,166	0,4225	0,3927
γ_2	-0,037	0,3885	-0,094

R^2 - 0,9252

As one can see, α and β_1 have positive sign which is in line with model predictions. That is, with the positive dynamics of cumulative demand, or with reduction of employment below its optimum level, the total number of employed people has a tendency to increase.

By consecutive elimination of insignificant variables we come to the following restricted model,

$$\Delta Et = \alpha(E^*_{t-1} - E_{t-1}) + \beta_1 \Delta Y + \beta_2 \Delta \pi_{t-1} + \varepsilon_t \qquad (8)$$

Estimation results are given in table 1.2.

Table 1.2

	Coefficient	Standard error	t – stat
α	0,1858	0,0651	2,8513
β_1	0,0439	0,0084	5,2216
β_2	-0,018	0,0061	-3,014

R2 - 0,9177

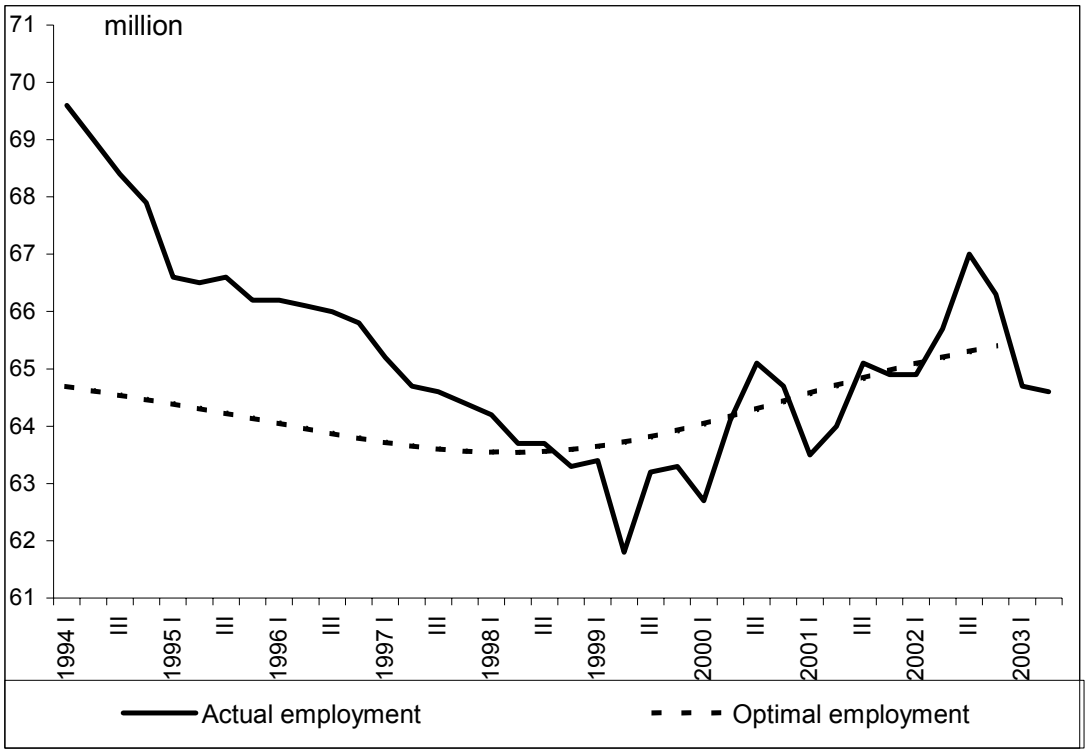
As one can see, following the restriction, the signs of α and β do not change, with substantial growth of significance, and practically the same R^2. Therefore, as a final model of employment we use equation 8.

The negative sign of inflation coefficient in the final model somewhat contradicts standard economic theory, since it implies positive slope of the Phillips curve. This result is probably associated with the negative influence of unstable inflation, which is usually associated with high inflation. The possibility of such discrepancy between theory and practice, given deep differences between inflation regimes, is mentioned by Friedman, in his Nobel lecture [9].

The values of the optimal employment level E_1^* ,…, E_T^*, are shown on the fig. 1. As one can see, estimated values of optimal and actual employment in the Russian economy had been gradually falling since the beginning of reforms, reaching absolute minima in 1998 and 1999, accordingly.

Fig. 1.

Changes of optimal and actual employment in Russian economy
(millions)



Source: actual level – Goskomstat of the Russian Federation, optimal level - Institute for Open Economy

Major reason for such development is a sharp reduction of labor demand in Russia caused by a reduction of country's cumulative demand (GDP). At the same time, according to the results of this research, up until the end of 1999 the actual employment was essentially higher than its optimal level. This result provides some background for the opinions of overemployment [1, 4] in the Russian economy at the beginning of reforms. The gap between the two parameters had been gradually falling and practically disappears by mid-1999. After this, the actual employment in Russian economy starts to closely follow its optimal level.

3.2. Optimal size of the labor force

The results of the estimation of the labor force model given by equations 6 and 7, are given in table 2.

$$A_t = A_t^* + \eta \Delta POP_t + \mu \Delta w_t + \varepsilon_t$$

Table 2

	Coefficient	Standard error	t – stat
η	-0,001282	0,00133	-0,00128
μ	0,000343	0,00589	0,05818

R^2 - 0,6871

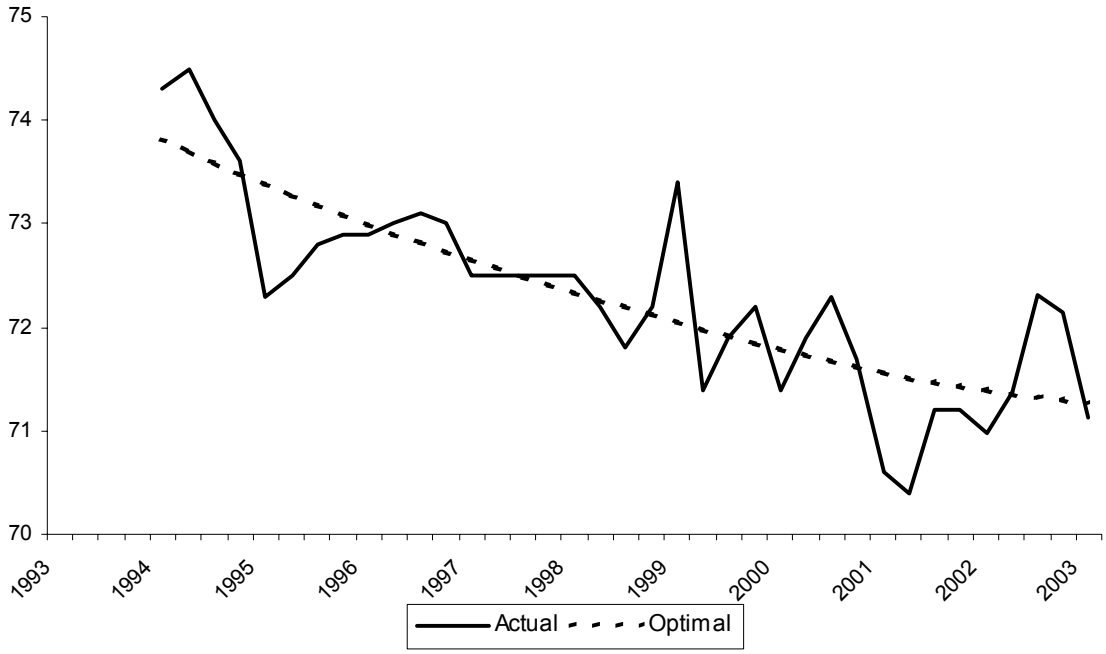
As one can see, μ has a positive sign, i.e. the growth of real incomes brings about the labor force increase, which is in line with the initial assumptions of the model. Whereas η has a negative sign, which contradicts the model. At the same time, both coefficients are statistically insignificant and close to zero.

By consecutive elimination of statistically insignificant coefficients we come to the conclusion that the optimum size of the labor force is a simple moving average of actual labor force. Such result is not surprising, given the fact, that the size of the labor force is a very static variable, not readily influenced by external factors.

The resulting values of the optimal size of the labor force are given in fig. 2. As one can see, the optimal size of the labor force had been steadily falling over the whole period.

Fig. 2.

Changes of optimal and actual size of Russian labor force
(millions)



Source: actual level – Goskomstat of the Russian Federation, optimal level - Institute for Open Economy

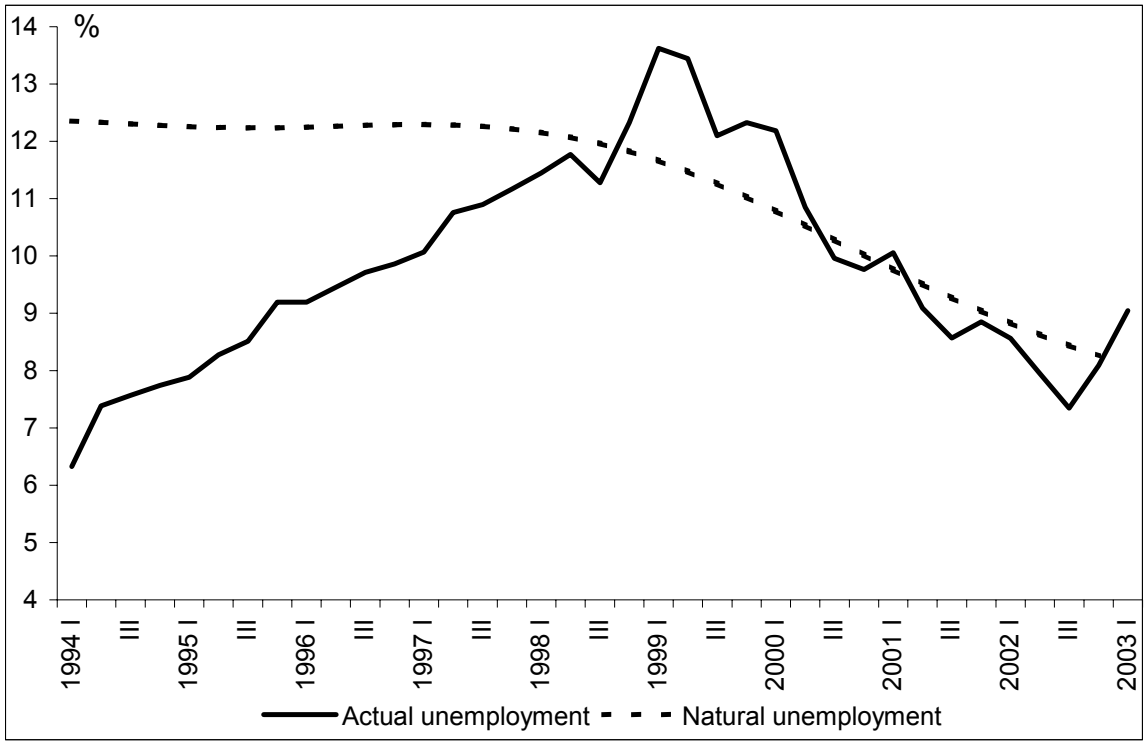
3.3. Natural rate of unemployment

A natural rate of unemployment is calculated using standard formula – a number of unemployed as the percentage of the labor force. Here, the optimal number of unemployed is a difference between optimal size of the labor force and the optimal employment. The results are shown in Figure 3.

One of the major results is that since the beginning of period and up to the end of 1998 the natural unemployment rate remained on a rather high, and remarkably stable level, varying between 12-12,5% of total labor force. This circumstance provides some background for the presence of a situation on the Russian labor market, which Kapelyushnikov [1] describes as an «adaptation without restructuring», or preservation of inefficient structure of employment by development of a network of informal inter-relationships between an employer and an employee.

Fig. 3.

Dynamics of natural and actual rates of unemployment in Russia
(% of the labor force)



Source: actual level – Goskomstat of the Russian Federation, optimal level - Institute for Open Economy

In particular, before 1998, because of the widespread use of such forms of firm-worker relationships as the massive wage arrears and the administrative leaves, the formal institutes of the labor market lost their function to be uniform and obligatory «game rules».

First of all, this allowed to soften initial adaptation to the rules of market economy management. However, such uncertainty in firm-worker relations had also

opened wide opportunities for preservation of disproportions in the labor market remained from the planned economy. Primarily, this uncertainty gave a lot of opportunities for numerous inefficient enterprises and helped to preserve employment in the depressive sectors of the economy and regions of the country. Under other conditions these regions would be forced to a more radical reduction of employment. The existence and propagation of this uncertainty has resulted in a formation of such high natural unemployment rate.

At the same time, starting from 1998 the natural unemployment rate acquired clear negative trend, reaching a level of 8,1% by the end of 2002. Such dynamics are primarily caused by overall recovery of the national economy. Nevertheless, the tendency of the reduction in the natural unemployment rate suggests that the crisis of 1998 initiated massive optimization of employment size and structure. Consequent improvement of financial condition among the majority of the employers allowed to speed up these processes.

As a result, since 1999 there had been a massive reallocation of labor resources towards successful sectors and enterprises, which has resulted in a reduction of the natural unemployment rate by 4 percentage points by the end of 2003.

Another, rather unexpected result is that until 1998 the actual rate of unemployment was much lower than its natural level, and since 2000 practically equals it. According to the definitions of various types of unemployment, this result shows that major constituent of general unemployment in Russia is structural rather than cyclical unemployment. In fact, more or less significant levels of cyclical unemployment in Russia was observed only after the 1998 financial crisis and practically disappeared by mid-2000. This result contradicts widespread academic opinion (2, 6) on primarily cyclical character of unemployment in Russia.

In the traditional area of applying the concept of natural unemployment rate, i.e. for determination of monetary and fiscal policy measures, such result allows to make a conclusion, that at the current moment, further softening of monetary and fiscal policy will not cause reduction of unemployment in Russia, since the actual rate of unemployment today practically equals its natural level.

At the same time, the most effective policy set for the reduction of unemployment is not in creation of new jobs as such, (i.e. investment climate improvement, investments promotion etc.), but rather measures aimed at smoothing out the existing structural disproportions of the Russian labor market.

One of such measures is to increase the efficiency of labor utilization by labor redistribution towards labor deficient regions and sectors of the economy through the improvement of geographical and social labor mobility.

To illustrate the main idea of this measure we provide rate dynamics in the regions with the lowest and highest unemployment levels (Tables 3 and 4).

As one can see from Table 3, in almost all regions with the lowest unemployment rate in 2003 the situation in the labor market continued to improve (rate of

unemployment has decreased in comparison with 2001). At the same time, in 4 of 10 regions with the highest unemployment rate, over the same period the problem just got worse.

Similar dynamics and unemployment rate gaps can be observed in comparison of closely located regions. For example, in the Central Federal District, with a reduction of unemployment in the Moscow oblast from 5,5 % in 2001, to just 3,6 %, in 2003, unemployment rate in neighboring Smolensk oblast has grown from 9,9 % to 12,9% over the same period. In North-Western Federal District, given the reduction of unemployment in St. Petersburg from 3,9% to 3,6%, unemployment in the surrounding Leningrad oblast has grown (from 6,9% to 7%) despite robust economic growth.

Table 3
10 regions with the lowest unemployment in 2003.

Rank	Region	2001	2003	Change since 2001 r.	2001 rank
1	Moscow	2,1	1,4	-0,7	1
2	Evenkia	2,9	2,6	-0,3	2
3	St.Petersburg	3,9	3,6	-0,3	3
4	Moscow oblast	5,5	3,8	-1,7	5
5	Yroslavskaya oblast	7,1	4,2	-2,9	16
6	Chukotsky A.O.	7,4	4,7	-2,7	20
7	Tverskaya oblast	7,8	5	-2,8	24
8	Lipetsk oblast	6,6	5,1	-1,5	13
9	Tulskaya oblast	5,2	5,2	0	4
10	Kostromskaya oblast	6	5,7	-0,3	7

Table 4
10 regions with the highest unemployment in 2003.

Rank	Region	2001	2003	Change since 2001 r.	2001 rank
79	Buryatia	18,5	13,5	-5	82
80	Karachaevo-Cherkessia	18,6	14,5	-4,1	83
81	Mariy-El	9,4	15	5,6	36
82	Adygeya	14,1	15,3	1,2	75
83	Kalmykia	19,1	17,4	-1,7	84
84	Aginsky-Buryat A.O.	23	18,7	-4,3	85
85	Tuva	23,9	19,4	-4,5	86
86	Dagestan	28,8	21,8	-7	87
87	Kabardino-Balkaria	16,8	22,5	5,7	80
88	Ingushetia	34,9	45,2	10,3	88

Source: Goskomstat of Russian Federation

The problem can be also seen on intraregional labor markets. In the majority of Russian regions there is one or few centers of economic activity with more or less favorable labor market conditions. At the same time, population living outside of such centers is forced to choose between minimum number of employers (primarily state funded organizations) or involved in a subsistence agricultural production.

This situation shows the importance of restrictions to labor mobility in Russia, which renders strong influence on Russian labor market. In particular, today one can speak of presence of labor deficit in several industrial regions of the country (first of all Moscow and St. Petersburg), which can hinder further development by increasing labor costs. Under these conditions, the creation of additional incentives by softening monetary or fiscal policies will most likely result in further growth in labor market disproportions, with limited influence on unemployment rate.

Besides, the result of disproportionately low rate of unemployment in Russia until 1998 can also provide some background for the discussions about the effectiveness and validity of monetary and fiscal policies over that period, or reasons for the generally higher inflation rates during 1990's. However, considering the amount of required analysis, we leave these questions as themes for further research.

4. Conclusion

In this research we estimated natural unemployment rate in the Russian Federation in 1994 - 2003. Considering versatile and complex character of factors, which influenced the labor market over that period, we estimated the natural rate by consecutive estimation of its major constituents, i.e. optimal employment and optimal size of the labor force. The estimation was made using modified Hodrick-Prescott filter, which allows the estimation of moving average series weighed on the influence of external factors.

Our estimates confirm earlier findings on the presence of over-employment in Russian economy at the beginning of reforms. The results also show, that until 1998 the actual rate of unemployment was much lower than its natural level.

Such results contradict existing opinion on mainly cyclical type of general unemployment in Russia. According to our results, cyclical unemployment in more or less significant levels was observed only shortly after the 1998 financial crisis and practically disappeared by the middle of 2000. Today, the actual rate of unemployment is very close to its natural level, which implies that its major constituent is the structural unemployment.

This result has direct application for determination of government labor policies. In particular, based on our results, it is possible to make a conclusion about limited effectiveness of government fiscal and monetary policies in decreasing unemployment and job creation. Under present conditions of growing disproportions on the labor market, the most effective policy measures could be redistribution of available labor

force towards developing regions and sectors of the economy, by increasing social and geographical mobility of population. At the same time, additional development stimuli like softening of monetary or fiscal policy would only lead to an increase of a labor deficit problem in a few industrial centers of the country with limited influence on the general unemployment rate.

Mathematical appendix

Equations (1, 5 and 8) in a general view can be shown as:

$$Y = X\beta + Y^* + \varepsilon, \quad \text{(A.1)}$$

Where:

Y - depended variable,

X - matrix of explanatory variable,

β - coefficient vector,

ε - error,

Y^* - smoothly varying parameter.

As opposed to the objective function used for the method of least squares, the objective function in this case looks as follows:

$$V(\beta, Y^*) = \varepsilon^{|}\varepsilon + \lambda(D_2 Y^*)^{|}(D_2 Y^*), \quad \text{(A.2)}$$

Or

$$V(\beta, Y^*) = (Y - X\beta - Y^*)^{|}(Y - X\beta - Y^*) + \lambda(D_2 Y^*)^{|}(D_2 Y^*) \quad \text{(A.3)}$$

Where λ is a "smoothness" parameter for a Y^* series, D_2 – difference operator, or the following matrix:

$$\begin{pmatrix} 1 & -2 & 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & -2 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 1 & -2 & 1 & 0 \\ 0 & \dots & 0 & 0 & 1 & -2 & 1 \end{pmatrix}$$

This objective function is an extension for Hodrick-Prescott objective function [15] in the case of existence of explanatory variable. As a result, for any value of β, vector Y^* is determined by HP filtering $(Y - X\hat{\beta})$ series, with smoothing parameter λ, or

$$\hat{Y}^* = HP^{-1}(Y - X\hat{\beta}), \quad \text{(A.4)}$$

Where HP^{-1} is smoothing operator, or a matrix inverse to:

$$HP = \lambda D_2^{|} D_2 + E, \quad \text{(A.5)}$$

Where E - identity matrix.

Substituting (A.4) into (A.3) and simplifying, we get:

$$V(\beta) = [(E - HP^{-1})(Y - X\beta)]^{|} \cdot [(E - HP^{-1})(Y - X\beta)] + \\ + \lambda[D_2 HP^{-1}(Y - X\beta)]^{|} \cdot [[D_2 HP^{-1}(Y - X\beta)] \quad \text{(A.6)}$$

Taking into account equation (A.5) we get:

$$V(\beta) = (Y - X\beta)^{|}[(E - HP^{-1})(E - HP^{-1}) + HP^{-1}(HP - E)HP^{-1}](Y - X\beta) =$$
$$= (Y - X\beta)^{|}[E - HP^{-1} - (E - HP^{-1})HP^{-1} + (E - HP^{-1})HP^{-1}](Y - X\beta) =$$
$$= (Y - X\beta)^{|}[E - HP^{-1}](Y - X\beta) \quad \text{(A.7)}$$

As one can see, in this function there is no *Y**, which allows us to minimize objective function, differentiating it to β. Therefore, the first order condition is:

$$\frac{dV(\beta)}{d\beta} = -2X^{|}(E - HP^{-1})X\beta + 2X^{|}(E - HP^{-1})Y = 0 \quad \text{(A.8)}$$

where vector of factors β can be estimated as:

$$\hat{\beta} = [X^{|}(E - HP^{-1})X]^{-1}X^{|}(E - HP^{-1})Y \quad \text{(A.9)}$$

Using (A.1) and (A.4) it is possible to get the equation for the error term:

$$\hat{\varepsilon} = (E - HP^{-1})(Y - X\hat{\beta}) \quad \text{(A.10)}$$

Whereas covariance matrix looks like:

$$\hat{\text{cov}}[\beta] = (X^{|}(E - HP^{-1})X)^{-1}X^{|}(E - HP^{-1})\hat{\sigma}_{\varepsilon}^{2}(E - HP^{-1})X(X^{|}(E - HP^{-1})X)^{-1} \quad \text{(A.11)}$$

References:

1. Friedman, Milton "The Role of Monetary Policy." *American Economic Review* 5 (March 1968): 1-17.
2. Friedman, Milton " Nobel Lecture: Inflation and Unemployment." Journal of Political Economy, vol. 85, no. 3, 1977.
3. Gordon, Robert J., "Price Inertia and Ineffectiveness in the United States," *Journal of Political Economy*, December, 1982, 90, 1087-117
4. Congressional Budget Office, "CBO's Method for Estimating Potential Output," *CBO Memorandum*, October, 1995.
5. Economic Planning Agency, *Keizai Hakusho Fiscal 2000*, 2000
6. Eisner R., "Opening Up the Growth Debate," *Wall Street Journal*, September 25, 1995, A18:3.
7. Hirose, Y., and K. Kamada, "A New Technique for Simultaneous Estimation of the Output Gap and Phillips Curve," Bank of Japan Working Paper Series, Working Paper 01-7, October, 2001.

8. Hodrick, R.J., and E.C. Prescott, "Postwar U.S. Business Cycles: An Empirical Investigation," *Journal of Money, Credit, and Banking*, Vol.29, No. 1, 1997, pp.1-16.
9. Kamada, R., and K. Masuda, "Effects of Measurement Error on the Output Gap in Japan," *Money and Economic Studies*, Vol. 19, No. 2, 2001, pp.109-154.
10. Okun, A., "Potential GDP: Its Measurement and Significance," *In Proceedings of the Business and Economics Statistics Section, American Statistical Association*, pp. 98-103.
11. Phelps, E.S. "Phillips Curve, Expectations of Inflation and Output Unemployment over Time." Economica, n.s. 34 (August, 1967): 254-81.
12. Phelps, E.S. "Money Wage Dynamics and Labor Market Equilibrium." In Microeconomic Foundations of Employment and Inflation Theory, edited by E.S. Phelps. New York: Norton, 1970.
13. Phillips, A.W., "The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957," *Economica*, No. 25, 1958, pp. 283-99.
14. Steiger, Douglas, J. H. Stock, and M. W. Watson, "The NAIRU, Unemployment and Monetary Policy," *Journal of Economic Perspectives* – Vol. 11, No. 1 – Winter 1997, pp. 33-49.
15. Tootell, G., "Restructuring the NAIRU, and the Phillips Curve," *New England Economic Review of the Federal Reserve Bank of Boston*, September/October 1994, 32-44.
16. Weiner, S., "New Estimates of the Natural Rate of Unemployment," *Economic Review of the Federal Reserve Bank of Kansas City*, Fall 1993, 78, 53-69.

17. Weiner, S., “The Natural Rate and Inflationary Pressures,” *Economic Review of the Federal Reserve Bank of Kansas City*, Summer 1994, 79, 5-9.
18. Капелюшников Р.И., Российский рынок труда: адаптация без реструктуризации. – М.: ГУ ВШЭ, 2001. – 309 с.
19. Коровкин А.Г., Динамика занятости и рынка труда: вопросы макроэкономического анализа и прогнозирования. – М.: МАКС Пресс, 2001. –320 с.
20. Коровкин А.Г., Движение трудовых ресурсов: анализ и прогнозирование. М.: Наука, 1990.
21. Оцу С. Советский рынок труда. М.: Прогресс, 1992.
22. Государственная и корпоративная политика занятости / Под ред. Т. Малевой; Моск. Центр Карнеги. - М., декабрь 1998. - 179 с.
23. Социальные ресурсы и социальная политика. Отв. Ред. академик С.С. Шаталин, к.э.н. В.Г. Гребенников. М.: Наука, 1990.
24. Экономика труда: (социально-трудовые отношения) / Под ред. Н.А. Волгина, Ю.Г. Одегова. – М.: Издательство «Экзамен», 2003. – 736с.